STABILITY OF QUARK MATTER AND QUARK STARS

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We use the confining chromodielectric model to study quark matter. In the version of the model with a quartic potential for the confining field we obtain two solutions for the mean-field equations. The resulting equations of state for strange quark matter are degenerate at large densities with a small energy barrier between them. As the density decreases the barrier gets higher lowering the probability for transitions between the two solutions. The solution that is lower in energy saturates around the nuclear matter density and corresponds to a chiral broken phase with massive quarks. The other solution, saturates at around five times the nuclear matter saturation density and corresponds to a chiral symmetric phase with massless quarks. Using the Tolman-Oppenheimer-Volko equations we study the structure of compact objects emerging from the equations of state. It turns out that the metastable equations yield pure quark stars with roughly a solar mass, and radii below 10 km. These objects, however, are not absolutely stable.

1. Introduction

According to the so-called strange matter hypothesis, formulated by Witten\(^1\), strange quark matter could be the ground state of strongly in-

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teracting matter. The energy per baryon number of such matter would be lower in energy than $^{56}$Fe. The strange matter hypothesis means that a nucleus would be a metastable state and could convert into a drop of strange quark matter (a strangelet).

Because of energy limitation of present day accelerators, they cannot provide direct experimental information on quark matter. However, recently, a number of different analysis of observational data suggest the possibility for the existence of compact objects with very small radii. Such compact objects cannot be easily accommodated in conventional models of baryon matter since their smallness requires a much softer equation of state (EOS) than those provided by the models. A rather obvious alternative is to consider that those small compact objects are made out of quarks. For such quark stars to exist, their density at the surface should be well above the nuclear matter saturation density. Taken Witten hypothesis for granted, quark stars would be an ultrastable bulk of matter.

From the theoretical point of view, this kind of issues should be ultimately addressed by quantum chromodynamics (QCD) because it is the theory for the strong interactions. It is well known, however, that it is a prohibitively complicated theory at the hadronic energy scales, due to its non-linearities. Alternative approaches are therefore in place. Essentially they consist on phenomenological descriptions of baryons and quark matter. The approaches are based on effective models which incorporate as much as possible the properties of QCD.

A number of phenomenological models for the strong interactions at intermediate energies have been developed since QCD was proposed. The MIT bag was the first relativistic model to be considered as an effective theory of the nucleon. The model was object of numerous refinements, in particular the inclusion of chiral mesons to endow it with an important property of QCD in the light quark sector, namely chiral symmetry. One must say that the bag model, as well as various versions of soliton models, provided a reasonable description of the nucleon properties and also of its excitations.

These effective models, originally designed for the nucleon, involving quarks as fundamental dynamical fields, have also been successfully used to describe infinite quark matter. The resulting equations of state have been applied to investigate the structure of compact stars\textsuperscript{2-6}.

The chromodielectric model (CDM)\textsuperscript{7} is one of such models, providing a reasonable phenomenology for the nucleon\textsuperscript{8}. On the other hand it also allows us to obtain EOS’s for dense quark matter. In the baryon number one
sector of the model, it yields soliton solutions representing single baryons with three quarks dynamically confined by a scalar field, $\chi$, whose quanta can be assigned to $0^{++}$ glueballs. When it is applied to quark matter in two or three flavors the resulting EOS turns out to be relatively soft at large densities. It is therefore tempting to try to describe quark stars in the framework of this model.

With a quadratic potential for the $\chi$ field, the model was applied to describe the inner part of hybrid stars. The resulting masses lie in the range $1 - 2M_\odot$, with the radii of the order of 10 km or higher, and a hadron crust of about 2 km. In this work we consider an extension of the model used in Ref. 10, taking quartic instead of quadratic potentials. In addition to these structures, the quartic model predicts another type of compact objects made out of quarks only, smaller and denser than neutron stars. Interestingly enough, these stars are metastable, and therefore they may decay into hybrid stars. However, it turns out that for such processes to take place, a reasonably high energy barrier has to be transposed, and therefore they may live for long periods.

From the observational point of view, the recent discovery of X-ray sources, by the Hubble and Chandra telescopes, increased the plausibility that these sources might be strange quark stars. In particular, the compact objects RXJ1856.5-3754 and 3C58, with apparently small radii, do not show evidence of spectral lines or edge features, reinforcing the conjecture for the existence of stars made out of strange matter. The phenomenology of these objects seems to be compatible with the small and dense quark stars reported in this work, but one should be aware that the data for those compact objects might not be accurate enough and eventually they are neutron stars.

This contribution is organized as follows. In section 2 we present the chromodielectric model, underlying its most important properties. In section 3 we describe quark matter in the mean field approximation and, finally, in section 4 we show the results obtained in our study of compact objects using the EOS predicted by the model.

2. The Chromodielectric model
The CDM contains quark and chiral meson degrees of freedom, in addition to a scalar-isoscalar chiral singlet chromodielectric field. The coupling of this field to fermions leads to quark confinement and this is an important
feature of the model. We write the CDM Lagrangian in the form

\[ L = L_q + L_{\sigma,\pi} + L_{q-\text{meson}} + L_{\chi}, \]

where

\[ L_q = i \bar{\psi} \gamma^\mu \partial_\mu \psi, \quad L_{\sigma,\pi} = \frac{i}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \tilde{\pi} \cdot \partial^\mu \tilde{\pi} - W(\tilde{\pi}, \sigma), \]

and \( W(\tilde{\pi}, \sigma) = (\lambda/4)(\tilde{\pi}^2 + \sigma^2 - f_\pi^2)^2 \) is the Mexican hat potential. In the \( u, d \) sector the quark-meson interaction is described by

\[ L_{q-\text{meson}} = \frac{\lambda}{2} \bar{\psi}(\partial + i \tilde{\tau} \gamma_5) \psi. \]

The current quark masses are zero but, due to the spontaneous chiral symmetry breaking enforced by the Mexican hat potential, the quarks acquire a dynamical mass.

The last term in (1) contains the kinetic and the potential piece for the \( \chi \)-field:

\[ L_\chi = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi). \]

The vector current is conserved for the Lagrangian (2), which is chiral SU(2) \( \times \) SU(2) symmetric. The vacuum expectation values of the chiral mesons are \( \langle 0|\pi|0 \rangle = 0 \) and \( \langle 0|\sigma_0|0 \rangle = f_\pi \). We work in the chiral limit, i.e. we take \( m_\pi = 0 \) for the pion mass. For the other parameters in the Mexican hat potential, which are kept fixed in our calculations, we take \( f_\pi = 0.093 \text{ GeV} \) for the pion decay constant, and \( m_\sigma = 1.2 \text{ GeV} \) for the sigma mass [in the Mexican hat, \( \lambda = m_\sigma^2/(2f_\pi^2) \)]. Besides the parameters in the potentials there is one more parameter in the model, namely the coupling constant, \( g \).

The potential term for the \( \chi \) field is

\[ U(\chi) = \frac{1}{2} M^2 \chi^2 \left[ 1 + \left( \frac{8 \eta^4}{\gamma^2} - 2 \right) \frac{\chi}{\gamma M} + \left( 1 - \frac{6 \eta^4}{\gamma^2} \right) \left( \frac{\chi^2}{(\gamma M)^2} \right) \right], \]

where \( M \) is the \( \chi \) mass. The parameterization used in (5) allows for a physically meaningful interpretation of the parameters \( \gamma \) and \( \eta \): \( U(\chi) \) has a global minimum at \( \chi = 0 \) and a local one at \( \chi = \gamma M \), and \( U(\gamma M) = \eta^4 M^4 \). The height of the local minimum, \( B = (\eta M)^4 \), is interpreted as a “bag pressure” and this is used to fix the parameters in \( U(\chi) \). Assuming the wide range \( 0.150 \leq B^{1/4} \leq 0.250 \text{ GeV} \), one has \( 0.08 \leq \eta \leq 0.15 \), using \( M = 1.7 \text{ GeV} \). We note that \( \gamma \) is not a free parameter since the quartic term of \( U(\chi) \) must be positive and the cubic term negative, which implies \( \gamma^2 \geq 6\eta^4 \). In the soliton sector of the model, best nucleon properties
are obtained for $G = \sqrt{gM} \sim 0.2$ GeV (only $G$ matters for the nucleon properties) and we keep that $G$ in our quark matter calculations.

In order to study strange quark matter, we add to the interaction Lagrangian (3) the term\(^{13}\)

$$\mathcal{L}_{s\text{-meson}} = \frac{g_s}{\chi} \bar{s} \gamma_s s,$$

accounting for the coupling between the strange quark and the $\chi$ field.

### 3. Quark matter in the mean field approximation

In the mean-field approximation, the pion field vanishes in homogeneous infinite matter. Therefore, the energy per unit volume of homogeneous $u,d$ quark matter interacting with sigma and chi is

$$E/V = \epsilon = \frac{N}{(2\pi)^3} \int_0^{k_F} 4\pi p^2 \sqrt{p^2 + M_q^2} dp + U(\chi) + W(\sigma) \quad (7)$$

where $N = 12 [2(\text{spin}) \times 2(\text{isospin}) \times 3(\text{color})]$ is the degeneracy factor and $k_F$ is the Fermi momentum related to the quark density, $\rho$, through

$$\rho = \frac{N}{6\pi^2} k_F^3 \quad \text{or} \quad k_F = \left( \frac{6\pi^2 \rho}{N} \right)^{1/3}, \quad (8)$$

and $M_q$ is the quark dynamical mass given by

$$M_q = \frac{g_\sigma}{\chi}. \quad (9)$$

Since the vacuum expectation value of chi is zero [$\chi = 0$ is the global minimum of the potential $U(\chi)$] it is clear that a vacuum in the Nambu–Jona-Lasinio sense cannot be defined in the present model. This was already anticipated in (7), where no negative energy particles have been considered.

From Eq. (7) one readily obtains variational equations for $\sigma$ and $\chi$. They are the following gap equations:

$$\frac{2\pi^2 M^2 \chi^4 D(\chi)}{Ng^2 \sigma^2} = \int_0^{k_F} \frac{p^2 dp}{\sqrt{p^2 + M_q^2}} \quad (10)$$

$$\frac{2\pi^2 \lambda (f_\pi^2 - \sigma^2) \chi^2}{Ng^2} = \int_0^{k_F} \frac{p^2 dp}{\sqrt{p^2 + M_q^2}} \quad (11)$$

where we have introduced the function

$$D(\chi) = 1 + \frac{3}{2} \left( \frac{8\eta^4}{\gamma^2} - 2 \right) \frac{\chi}{\gamma M} + 2 \left( 1 - \frac{6\eta^4}{\gamma^2} \right) \frac{\chi^2}{(\gamma M)^2}. \quad (12)$$
For quadratic potentials \((\gamma \to \infty)\) this function is \(D = 1\).

In order to study strange quark matter in beta equilibrium, an electron gas must also be considered. The mean-field energy per unit volume for strange quark matter in the CDM (plus electrons) is now given by

\[
\varepsilon = \frac{1}{\alpha} \sum_{f = u, d} \int_{0}^{k_{f}} \frac{d^{3}k}{(2\pi)^{3}} \sqrt{k^{2} + m_{f}(\sigma, \chi)^{2}} + \frac{1}{\alpha} \int_{0}^{k_{e}} \frac{d^{3}k}{(2\pi)^{3}} \sqrt{k^{2} + m_{s}(\chi)^{2}}
\]

\[+ 2 \int_{0}^{k_{e}} \frac{d^{3}k}{(2\pi)^{3}} \sqrt{k^{2} + m_{e}^{2}} + U(\chi) + \frac{m_{s}^{2}}{8f_{\pi}}(\sigma^{2} - f_{\pi}^{2})^{2}, \tag{13}\]

where the first two terms refer to quarks and the third one to the electrons, all described by plane waves. The degeneracy factor is now \(\alpha = 6\) (for spin and color). The last term is the Mexican hat potential (with \(\tilde{\eta} = 0\) and \(f_{\pi} = 93\) MeV). The \(k_{i}\) in (13) are the Fermi momenta of quarks and electrons.

The quark masses in (13) are \(m_{u, d} = g_{u, d}\frac{\sigma}{f_{\pi}}\) and \(m_{s} = g_{s}\frac{\chi}{\xi_{3}}\) with the coupling constants given by \(g_{u} = g(f_{\pi} + \xi_{3})\), \(g_{d} = g(f_{\pi} - \xi_{3})\) and \(g_{s} = g(2f_{K} - f_{\pi})\) \([\xi_{3} = -0.75\) MeV, \(f_{K} = 113\) MeV]. A variational principle applied to the energy density, Eq. (13), leads to two gap equations for \(\sigma\) and \(\chi\) of the type of eqs. (10) and (11).

As in the interior of a compact star the matter should satisfy both the electrical charge neutrality and chemical equilibrium we impose

\[\frac{2}{3} \rho_{u} - \frac{1}{3} \rho_{d} - \frac{1}{3} \rho_{s} - \rho_{e} = 0, \tag{14}\]

[here, \(\rho_{i} = \alpha k_{i}^{2}/(6\pi)^{2}\) stand for each flavor density] and

\[\mu_{d} = \mu_{u} + \mu_{e}, \quad \mu_{d} = \mu_{s}, \tag{15}\]

where \(\mu = \sqrt{m^{2} + k_{F}^{2}}\) is the chemical potential for each particle. These conditions should supplement the gap equations, and altogether we have a system of six algebraic equations to solve at each baryon density

\[\rho = \frac{1}{3}(\rho_{u} + \rho_{d} + \rho_{s}). \tag{16}\]

The solution of the system of equations are the meson fields, \(\sigma\) and \(\chi\), and the Fermi momenta, \(k_{u}, k_{d}, k_{s}\) and \(k_{e}\). For the same set of model parameters we found two stable solutions, hereafter denoted by I and II (for details see Ref. 14).

For both solutions \(\sigma\) is always close to \(f_{\pi}\). In solution I, the \(\chi\) field is a slowly increasing function of the density, remaining always smaller than \(\sim 0.05\) GeV. For such a small \(\chi\), the quartic potential and the quadratic
potential are indistinguishable, thus, in practice, solution I corresponds to the one obtained and used by Drago et al.\cite{10} in the framework of the quadratic potential. Due to the smallness of the $\chi$ field, quark masses are large and the system is in a chiral broken phase. The solution II exhibits a large confining field, $\chi \sim \gamma M$ (local minimum of $U$), independent of the density. The resulting quark masses are similar for the three flavors and very close to zero (chiral restored phase). Therefore, the chemical potentials in solution II are dominated by the Fermi momentum contribution, $\mu_u \simeq \mu_d \simeq \mu_s$ and $\mu_e \simeq 0$, i.e., in solution II there are almost no electrons. Besides solutions I and II, there is an additional unstable solution corresponding to $\chi \sim \gamma M/2$ (local maximum of $U(\chi)$).

The energy per baryon number as a function of the baryon density (EOS) is readily evaluated for each solution. For each solution we obtained the corresponding energy per baryon number as a function of the baryon density (EOS) (see Fig. 1). EOS-I is not sensitive to $\gamma$ and $\eta$ (since $\chi$ is small), just depends on $G$, and it is rather similar to the one used in Ref. 10. The saturation density occurs at a low density, slightly higher than the nuclear matter equilibrium density, $\rho_0$. Its shape, at intermediate densities, is similar to hadronic EOS's (see Ref. 9 for the two flavors sector).

The EOS-II is also insensitive to $\gamma$, but does depend on $\eta$ [in fact, the dependence is on $(\eta M)^4$, as we have already discussed]: the energy per baryon number increases with $\eta$ and so does the saturation density. Depending on $\eta$ the minimal energy per baryon number of solution II can be either below or above solution I. For $\eta \sim 0.12$ the saturation occurs at $\rho \sim 5\rho_0$ and the energy per baryon number is some 230 MeV higher than for solution I at its saturation density. The two stable solutions are almost degenerated at high densities in the narrow range $0.1 \leq \eta \leq 0.12$. In Fig. 1 the dotted lines refer to the EOS for the unstable solution of the gap equations (supplemented by electric neutrality and beta equilibrium conditions), for which the $\chi$ is at the local maximum of the potential (5). In order to undergo a transition from I to II, the system has to go through the energy barrier represented by the dotted EOS. The barrier gets higher at small densities and, for $\eta = 0.12$ (third panel in Fig. 1), at $\rho \sim 5\rho_0$, $\epsilon/\rho \sim 2.7$ GeV for the unstable solution. Therefore a transition from one regime to the other is not likely to occur and both minima in the EOS I and II are stable.

In a 3D plot of the energy per baryon number versus $(\rho, \chi)$ the stable solutions correspond to two distinct “valleys”, and the unstable solution mentioned above corresponds to the top of the barrier between the two
Figure 1. Energy per baryon number versus density for solutions I (solid line, small $\chi$) and II (dashed line, $\chi \sim \gamma M$) and various parameters $\eta$ (other model parameters: $M = 1.7$ GeV, $g = 0.023$ GeV and $\gamma = 0.2$). The dotted line corresponds to the unstable solution with $\chi \sim \gamma M/2$.

Figure 2. 3D plot of the energy per baryon number versus $(\rho, \chi)$. Valleys (see Fig 2).

Regarding energetics, both phases are almost degenerated at high densities and have similar shapes in the $P\times\epsilon$ plane even at intermediate densities.
(or energy densities) in the narrow range $0.1 \leq \eta \leq 0.12$. In that region of $\eta$, one solution is not clearly lower in energy than the other. However, we should point out that they correspond to two different $\chi$ values and for the system to undergo a transition from the chiral restored to the chiral broken phase it has to go through a high potential energy barrier.

4. Quark stars

In order to investigate the structure of stars, for both EOS, we solved the Tolman-Oppenheimer-Volko (TOV) equations:

$$\frac{dP}{dr} = \frac{[\epsilon(r) + P(r)]M(r) + 4\pi r^3 P(r)}{r^2 \left(1 - \frac{2M(r)}{r}\right)}, \quad \frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$

(17)

where $P(r) = \rho^2 \frac{d\rho}{d\rho} \left(\frac{\rho}{\rho}\right)$ is the pressure and

$$M(r) = \int_0^r 4\pi r^2 \epsilon(r) dr$$

(18)

is the mass contained in a sphere of radius $r$.

Since EOS-I is identical to the one using a quadratic potential, it leads to stars that have the same phenomenology as the hybrid stars obtained by Drago et al.\textsuperscript{10}: $R \sim 10 - 12$ km, a hadron crust and a mass $M \sim 1 - 2M_\odot$. At low densities, hadronization occurs and an hadronic equation of state should be used, replacing EOS-I.

The EOS-II saturates at a high density and, in addition, the system is not likely to undergo a transition to solution I, so that one should not perform any connection to the hadronic sector: the EOS-II alone generates a new family of strange quark stars. In Fig. 3 it is shown the mass-radius relation for different values of $\eta$. These quark stars are smaller and denser in comparison with those resulting from EOS-I. For $\eta \sim 0.115$ (and $M = 1.7$ GeV, yielding $B^{1/4} \sim 0.195$ GeV) one obtains a maximum radius $R \sim 6$ km and a corresponding mass $M \sim 0.9M_\odot$. According to our calculation, such star has a central density of $10\rho_0$ ($\rho_0$ is the nuclear matter density) and a central energy density $\epsilon \sim 3 \times 10^{15}$ g/cm$^3$. At the edge, the density drops to $5\rho_0$ and $\epsilon \sim 1.35 \times 10^{15}$ g/cm$^3$. The ratio $\epsilon/\rho$ remains approximately constant inside the star. From Fig. 3 one concludes that the mass-radius relation for these strange small stars mainly depends on the height of the local minimum of the $\chi$ potential.

These stars, as already discussed, are not absolutely stable and therefore they may decay into hybrid stars.
Figure 3. Mass versus radius for the pure quark stars (solution II) in the CDM model.

References