

# Air Resistance: Distinguishing Between Laminar and Turbulent Flow

## 0.1 Introduction

You have probably heard that objects fall (really, accelerate) at the same rate, independent of their mass. Galileo demonstrated this to be true by dropping two cannonballs, one having ten times the mass than the other, from the top of the Leaning Tower of Pisa. Why then does a rock fall so much faster than a feather? The usually neglected qualification to the law that all objects fall at the same rate is that the law is only true in a vacuum, when the object is falling in a space with no atmosphere and no *air resistance*. Since Earth most certainly has an atmosphere, the law does not strictly apply, and rocks and feathers are free to fall at different rates and not break the laws of physics.

Because the force due to air resistance is velocity dependent, Newton's laws are harder to analyze when air resistance is taken into account, so air resistance is usually seen as an annoyance. Air resistance also acts as a frictional force, slowing objects down; for example, air resistance makes cars less fuel efficient. But it can also be helpful in many situations. Without air resistance neither planes nor helicopters could fly. Throwing a boomerang would be like throwing a normal stick. When properly understood, air resistance can be used to our advantage. In this lab you will characterize the type of air resistance a falling balloon feels and decide whether the balloon is experiencing *laminar* or *turbulent* air flow.

In most introductions to air resistance, the flow of air around an object is assumed to be what is called laminar flow. Laminar flow is characterized by layers, or laminas, of air moving at the same velocity. In figure 1a, the lines of flow are parallel, smooth and laminas are not interacting. No air molecules are exchanged between the laminas, and the flow need not be in a straight line. Laminar flow is assumed when the relative speed of an object moving in air is low. "Low speed" is of course a relative term, and surprisingly, everyday objects often move through air in a non-laminar manner. In turbulent flow, the flow lines are disorganized, distributed in ever changing eddies, as shown in figure 1b. Air molecules of differing speeds bump into one another and their speeds are constantly changing.

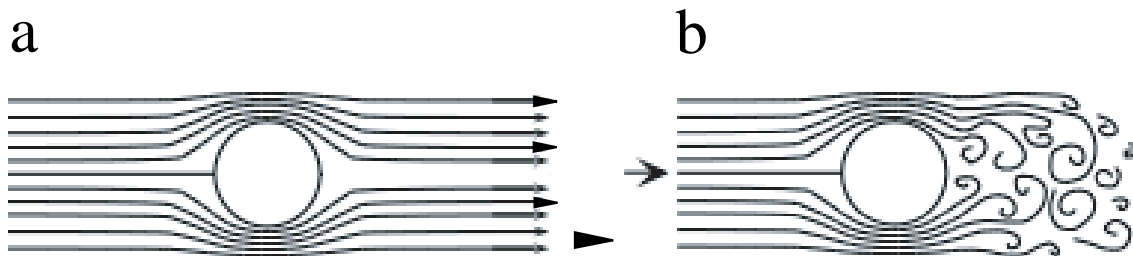


Figure 1: **a)** laminar flow consists of isolated layers of air with different velocities. **b)** Turbulent flow consists of unorganized motions of air molecules which often change their velocities, due to interactions with one another.

## 0.2 Theory

### Laminar and Turbulent Force Laws

It turns out that deriving a force law for a sphere in laminar flow is not an easy task; it requires an understanding of basic fluid mechanics, which could easily be a class in itself. The force on a sphere in laminar flow is called Stoke’s law and is given by

$$F = 6\pi\eta r v \tag{1}$$

where  $r$  is the radius of the sphere,  $v$  is the velocity of the sphere relative to the fluid and  $\eta$  is a property of the fluid called the viscosity. Viscosity is a kind of internal friction, the higher the viscosity the harder it is for the fluid to “flow”. For example, honey has a higher viscosity than water. Since the force is dependent on the viscosity, laminar flow is also called viscous flow. Notice the force is proportional to radius and velocity.

The exact force law for turbulent flow also requires a full fluid mechanics analysis but an expression describing what the force is proportional to (in this case cross-sectional area  $A$ , air density  $\rho$  and relative velocity  $v$ ) can be readily derived from the physics you have learned thus far.<sup>1</sup> The exact law is

$$F = \frac{1}{2}\rho C_d A v^2 = \frac{1}{2}\pi\rho C_d r^2 v^2 \tag{2}$$

where  $\rho$  is the density of air and  $C_d$  is the *drag coefficient*, which for a smooth sphere is about .47. In general,  $C_d$  is a function of the geometry and material make-up of the object. For example, if the sphere isn’t perfectly smooth,  $C_d$  would be higher than .47. Figure 2 shows some common shapes and car designs and their approximate  $C_d$  values.  $C_d$  is almost always experimentally determined in a wind tunnel.

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<sup>1</sup>If it interests you, try deriving a proportional force law by considering a sphere with mass  $M$  and cross-sectional area  $A$  moving through air at speed  $v$ . Assume that the density of air is  $\rho$ , with individual air molecules having mass  $m$ . Your expression should be in terms of  $A$ ,  $\rho$  and  $v$ . (Hint: Imagine the sphere sweeping out a volume in some time  $\Delta t$  (what is the volume?). How many air molecules are in that volume (this is the same number the sphere will displace)? Assume that each time the sphere displaces an air molecule, it speeds the air molecule up from rest to speed  $v$ , and exerts a force on that air molecule proportional to  $mv/\Delta t$ . Use  $F = ma$  to find the total force on the sphere.)






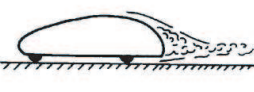









Shape	$C_d$	Shape	$C_d$	Shape	$C_d$
	1.17		0.38		0.8 - 0.9
	1.42		0.42		0.24
	1.42		0.59		0.35
	1.38		0.8		0.16
	1.05		0.5		0.13

Figure 2: Some common shapes and car designs and their drag coefficients. Notice how much larger the drag coefficient of the bus shape vehicle is than normal car designs.

### Falling in Air and the Reynolds Number

Now consider a sphere in free fall in air experiencing a resistive force dependent on velocity,  $F(v)$ . As the velocity of the sphere increases, so does the force on it, and the sphere eventually reaches a *terminal velocity*,  $v_T$ , and continues to fall at that velocity until it hits the ground. At this point, the force of gravity on the sphere and the air resistance force are equal, so

$$F(v_T) = Mg. \quad (3)$$

In particular, for laminar flow

$$6\pi\eta r v_T = Mg \quad (4)$$

and for turbulent flow,

$$\frac{1}{2}\rho C_d A v_T^2 = Mg. \quad (5)$$

Note in what manner  $M$  is proportional to both  $r$  and  $v$  for laminar and turbulent flow.

It is often desirable to predict which type of flow an object will be in before it is dropped or exposed to an air flow. A parameter called the *Reynolds number*,  $R_e$ , is often used to determine which type of flow an object is experiencing. The Reynolds number is proportional to the ratio of the turbulent force to the laminar force. The smooth, laminar regime generally occurs at a Reynolds number of one and below while the turbulent regime is in the Reynolds number range of  $10^3$  and above. In the case of a sphere,

$$R_e = \frac{\rho d v}{\eta} \quad (6)$$

where  $d$  is the diameter of the sphere.

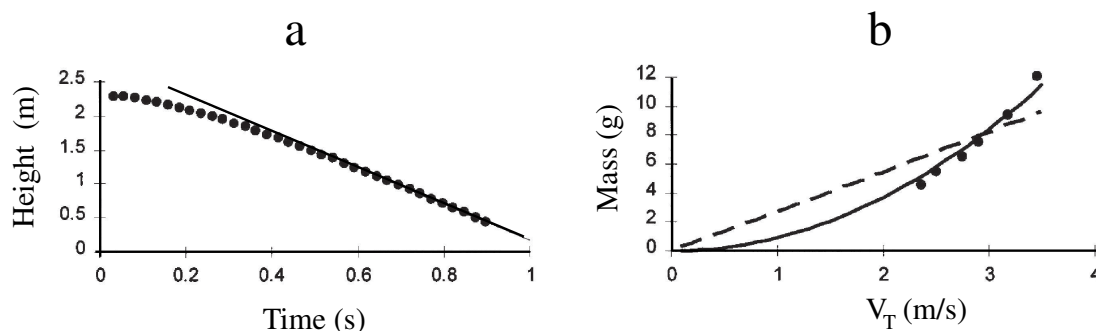


Figure 3: **a)** Distance versus time data for a balloon drop. The balloon accelerates at first but then drops at a constant velocity  $v_T$ . By fitting a line to the last several data points,  $v_T$  can be approximated as the slope of that line. **b)** Mass versus terminal velocity data and the best fit line and parabola which include the origin. This graph was created by compiling data from several different distance versus time drops. As can be seen, the parabola is a much better fit.

### 0.3 Experimental Method and Analysis

To study the nature of air resistance on a balloon, you will drop a balloon onto a motion sensor and record distance and time data for different drops. By varying the mass of the balloons you will be able to determine whether the balloon is experiencing laminar or turbulent flow. By varying the radii of the balloons you will be able to tell how the force on the balloon depends on radius. You will also be able to determine the Reynolds number  $R_e$  and drag coefficient  $C_d$  of the balloon.

Your lab write-up should summarize all of the investigations you undertook in the lab. Include all important findings and plots; motivate your arguments and discuss the method you used to come to your results. Be sure and discuss possible sources of random or systematic error.

#### Laminar or Turbulent Flow?

The idea in analyzing the data is to plot the mass  $M$  of the balloon versus the terminal velocity  $v_T$  of the balloon for several different balloon masses. By analyzing this plot, you should be able to tell whether the flow is more laminar or turbulent. From equation (4), laminar flow is characterized by both  $r$  and  $v$  being proportional to the mass of the balloon  $M$ . This means that if the balloon is experiencing laminar flow, when you plot  $M$  versus  $v_T$ , you should get a straight line. From equation (5), turbulent flow is characterized by the squares of both  $r$  and  $v$  being proportional to  $M$ . In this case, if you plotted  $M$  versus  $v_T$ , you should get a parabola.

You can vary the mass of the balloons by taping pennies to the bottom. Start with no pennies attached and add a penny for each drop. Don't tape too many on; remember, your balloon must reach terminal velocity at least .25 m or so before it hits the ground (or rather the cage with the motion detector in it).

You can get  $v_T$  by plotting distance versus time of the drop of the balloon. The plot should be basically parabolic, like free fall without air resistance, in the beginning and then straighten out in the end. The slope of this line at the end is  $v_T$  (see figure 3a). The balloon falls at a constant

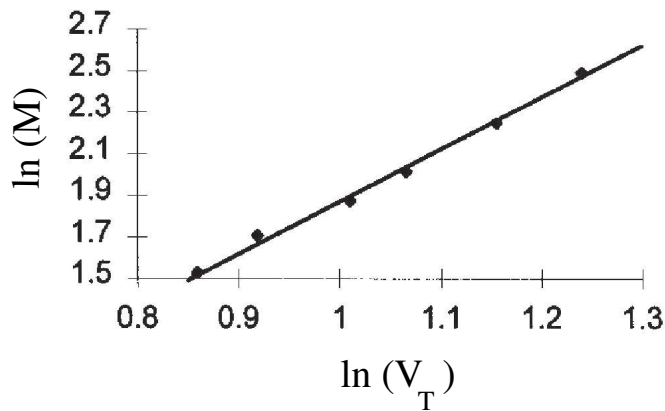


Figure 4: A log-log plot,  $\ln(M)$  versus  $\ln(v_T)$ . The slope  $n$  of the best fit line is the power which makes  $v^n$  proportional to  $M$ , as in equation (7).

speed because the air resistance force and the force of gravity have balanced out. You may wish to drop it from a ladder.

It is important to realize that your best fit line or curve needs to go through the origin (see figure 3b). This is because, as  $M$  gets small,  $v_T$  should also get small, since  $M$  is proportional to a positive power of  $v$  in both laminar and turbulent flow ((4) and (5)). Noting this, which models fits your data best?

An alternative way of analyzing the data is to note that you are looking for a power law relation in the form of

$$M \propto v^n. \quad (7)$$

The exponent  $n$  should be one if the sphere is in laminar flow and two if it is in turbulent flow. To figure out what value of  $n$  fits the data best you can plot the natural logarithm ( $\ln$ ) of  $M$  versus the natural logarithm of  $v_T^n$  (see figure 4). This type of plot is called a log-log plot, and is a good way to find the best value of  $n$  for the following reason. If you take the  $\ln$  of both sides of the equation  $M = Kv_T^n$ , where  $K$  is a constant, the resulting expression is

$$\ln(M) = \ln(Kv_T^n) = \ln(K) + \ln(v_T^n) = n \ln(v) + \ln(K). \quad (8)$$

This changes the expression so that  $\ln(M)$  is linearly proportional to  $\ln(v)$ , with  $n$  as the slope of the line and  $\ln(K)$  as the y-intercept (log-log plots in this lab do not necessarily have to include the origin). Fitting a best-fit line to the plot, and determining the slope of that line, gives the best value of  $n$  for your data. Using this technique, find the best value of  $n$  for your data.

### Radius Dependence

Hopefully you have discovered the turbulent flow model in which  $M \propto v^2$  fits the data much better than the laminar flow model in which  $M \propto v$ . You can now test whether the turbulent model correctly predicts the dependence on radius  $r$  of the balloons by analyzing how the quantity  $M/v_T^2$  depends on  $r$ . Note that you have to determine  $M$  and  $v_T$  for each balloon to make this

comparison. The number of pennies you attach isn't too important as long as you get a good value for  $v_T$ .

Plot  $M/v_T^2$  versus  $r$  for several different balloons with differing radii. Does  $M/v_T^2$  depend linearly or quadratically on  $r$ ? Remember, since  $M/v_T^2$  is proportional to a positive value of  $r$ , just as  $M$  is proportional to  $v$ , the best fit line or curve should go through the origin. Now get the best value of  $n$  that fits your data by making a log-log plot of these quantities and fitting the data to a best-fit line. How close is the value to two?

### The Reynolds number and drag coefficient

As discussed before, the value of the Reynolds number  $R_e$  is a good indicator for what type of flow the balloon experiences. From equation (6)

$$R_e = \frac{\rho d v_T}{\eta}. \quad (9)$$

The density of air is  $\rho = 1.28 \text{ kgm}^{-3}$  and the viscosity of air is  $\eta = 1.83 \times 10^{-5} \text{ Nsm}^{-2}$ . Approximately, what is  $R_e$  for your balloons (use an average value for  $v_T$ )? Using this result, which type of flow would you predict the balloons are in?

Another parameter you can determine is the drag coefficient  $C_d$ , which as stated before, should be .47 for a sphere. The simplest way to do this is to use the log-log plot you made when determining that  $M \propto v^2$ . Remember, a sphere in turbulent flow at speed  $v_T$  experiences a force upward equal to the gravitational force downward, so  $\frac{1}{2}\rho C_d A v_T^2 = Mg$ . Taking the natural log of both sides, and noting equation (8), we get

$$\ln(M) = 2 \ln(v_T) + \ln\left(\frac{\rho C_d A}{2g}\right). \quad (10)$$

This shows that the y-intercept  $\ln(K)$  of your best-fit line in your  $\ln(M)$  versus  $\ln(v_T^2)$  plot is equal to  $\ln\left(\frac{\rho C_d A}{2g}\right)$ . Use this to determine  $C_d$  for the balloons. Is it close to the expected value for a sphere?